LEZIONI DI MATEMATICA

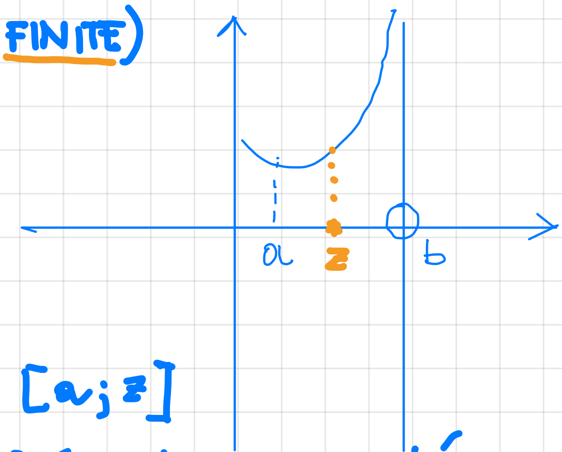
INTEGRALI IMPROPRI

$$\dots\dots\; \int_a^z f(x)\,dx$$

CLASSE : V LICEO SCIENTIFICO - LEZIONE: N. M5041

MARCO BRAICO

INTEGRALI IMPROPRI (DISCONTINUITÀ FINITE)



SIA $f(x)$ definita in $[a;b[$

$\exists\, f(a)$, $\nexists\, f(b)$

$z \in [a,b[$

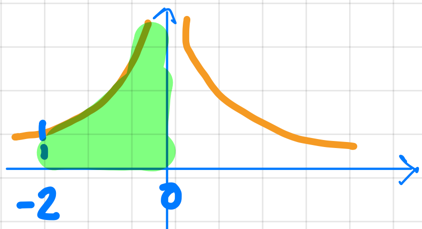
definisco la FUNZIONE INTEGRALE in $[a;z]$

$F(z) = \int_a^z f(x)\,dx$ in $[a,z]$ f è continua perché $z \neq b$

se $\exists$ finito $\lim_{z\to b^-} F(z) \to$ ALLORA L'INTEGRALE

$$\int_a^b f(x)\,dx = \lim_{z\to b^-} \int_a^z f(x)\,dx \quad \text{CONVERGE}$$

ES: $f(x) = \dfrac{1}{\sqrt[3]{x^2}}$ in $[-2;0]$



DOMINIO $x \neq 0$

$$\int_{-2}^{0} f(x)\,dx = \lim_{z\to 0^-} \int_{-2}^{z} \frac{1}{\sqrt[3]{x^2}}\,dx = \lim_{z\to 0^-} \int_{-2}^{z} x^{-\frac{2}{3}}\,dx =$$

$$\lim_{z\to 0^-}\left[\frac{1}{-\frac{2}{3}+1}\, x^{-\frac{2}{3}+1}\right]_{-2}^{z} = \lim_{z\to 0^-}\left[\frac{3}{1}\, x^{\frac{1}{3}}\right]_{-2}^{z} = \lim_{z\to 0^-} 3\left(z^{\frac{1}{3}} - (-2)^{\frac{1}{3}}\right)$$

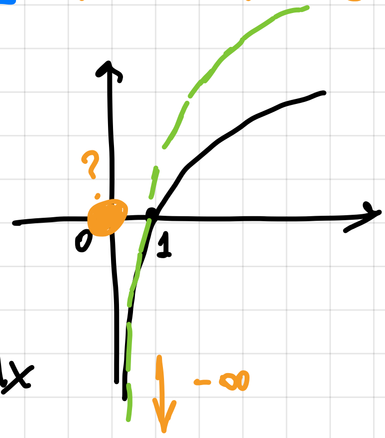
$$= \lim_{z\to 0^-} 3\left[\sqrt[3]{z} - \sqrt[3]{(-2)}\right] = \lim_{z\to 0^-} \underbrace{\cancel{3\sqrt[3]{z}}}_{\downarrow\, 0} + 3\sqrt[3]{2} = \boxed{3\sqrt[3]{2}}$$

L'INTEGRALE CONVERGE A UN VALORE FINITO

ESERCIZIO

$$\int_0^1 2\ln x\, dx$$

DOMINIO



$u = \ln x$

$u' = \frac{1}{x}$

$v' = 1$

$v = x$

$$F(z) = \int_z^1 2\ln x\, dx = 2\int_z^1 \underbrace{\ln x}_{u}\, \underbrace{dx}_{v'}$$

PE PARTI :

$$uv - \int_z^1 v\, u' \rightarrow x\ln x - \int_z^1 \cancel{x}\cdot\frac{1}{\cancel{x}}\cdot dx$$

$$\left[x\ln x - x\right]_z^1 = (1\cdot\cancel{\ln 1} - 1) - (z\ln z - z) =$$

$$= \boxed{-1 - z\ln z + z} \qquad F(z)$$

$$\lim_{z\to 0^+}\left(-1 - \underbrace{z\ln z}_{0\cdot(+\infty)} \cancel{+z}\right) = \underbrace{0\cdot \ln z}_{0} \rightarrow 0$$

ORDINI DI INFINITO

$$= \boxed{-1} \rightarrow 2(-1) = \boxed{-2}$$

LEZIONI DI MATEMATICA

INTEGRALI IMPROPRI IN UN INTERVALLO ILLIMITATO

CLASSE : V LICEO SCIENTIFICO - LEZIONE: N. M5042

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$[a, +\infty[$

$$\int_a^{+\infty} f(x)\,dx = \lim_{z \to +\infty} \int_a^{z} f(x)\,dx$$

CONVERGE finito

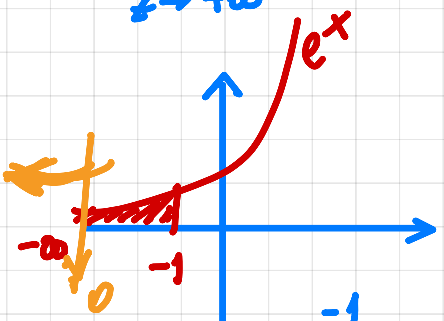
DIVERGENTE infinito

NON DETERMINATO $\nexists$ limite

- $$\int_1^{+\infty} \frac{1}{x^2}\,dx = \lim_{z \to +\infty} \int_1^{z} x^{-2}\,dx = \lim_{z \to +\infty} \left[-\frac{1}{x}\right]_1^{z} =$$

$$\lim_{z \to +\infty} \left[-\frac{1}{z} + 1\right] = 1$$

$\frac{1}{z} \to 0$



- $$\int_{-\infty}^{-1} e^x\,dx = \lim_{z \to -\infty} \int_z^{-1} e^x\,dx = \lim_{z \to -\infty} \left[e^x\right]_z^{-1} =$$

$$= \lim_{z \to -\infty} \left(e^{-1} - e^{z}\right) = \frac{1}{e} - 0 = \frac{1}{e}$$

••) $\displaystyle\int_{-\infty}^{0} \frac{e^x}{\sqrt{1-e^{2x}}}\,dx = \lim_{z\to-\infty} \int_{z}^{0} \frac{e^x}{\sqrt{1-e^{2x}}}\,dx$

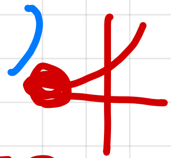
sostituzione $e^x = t \to x = \ln t \to dx = \frac{1}{t}\,dt$

$$\left.\begin{array}{ll} x\to z & t\to e^z \\ x\to 0 & t\to e^0 = 1 \end{array}\right\} \lim_{z\to-\infty} \int_{e^z}^{1} \frac{\cancel{t}}{\sqrt{1-t^2}}\cdot\frac{1}{\cancel{t}}\,dt =$$

$$\lim_{z\to-\infty} \left(\arcsin t\right)_{e^z}^{1} = \lim_{z\to-\infty}\left(\arcsin 1 - \arcsin e^z\right)$$

$\downarrow$

$\arcsin e^{-\infty} = 0$



$\arcsin 1 = \boxed{\frac{\pi}{2}}$