

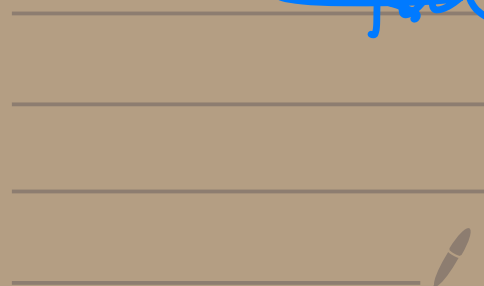


LEZIONI DI MATEMATICA

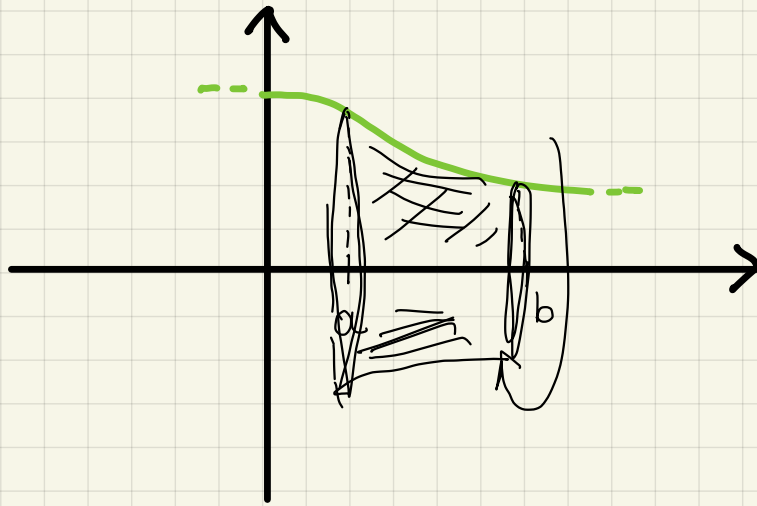
IL CALCOLO DEI VOLUMI CON GLI INTEGRALI

CLASSE : V LICEO SCIENTIFICO - LEZIONE: N. **M5039**

MARCO BRAICO

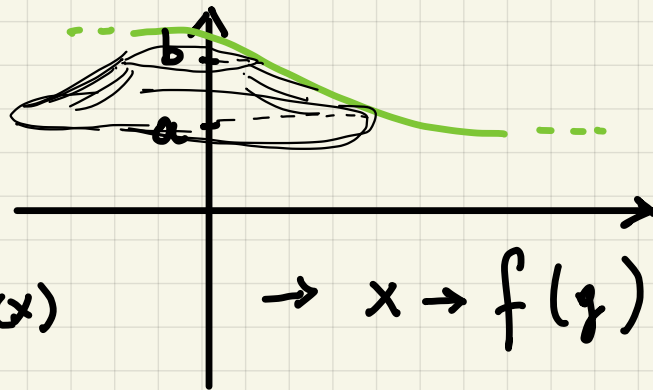


•) RUOTA INTORNO A x



$$\pi \int_a^b [f(x)]^2 dx$$

•) RUOTA INTORNO A y

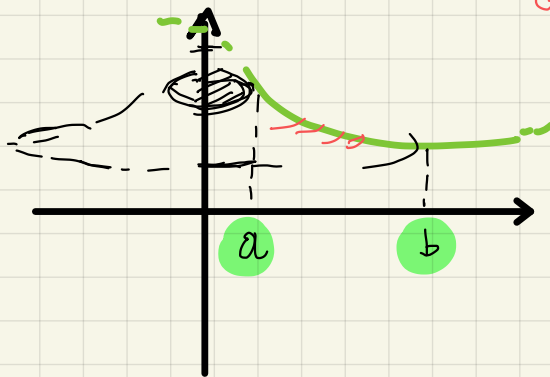


$$f^{-1} : y = f(x) \rightarrow x \rightarrow f(y)$$

$$y = x^2 \rightarrow x = \sqrt{y} \rightarrow V = \pi \int_a^b f(y) dy$$

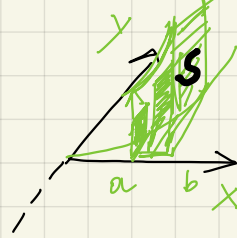
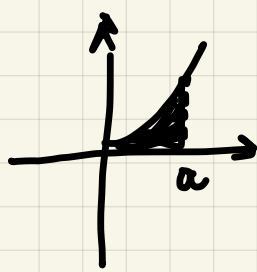
•)

GUSCI CILINDRICI



$$V = \pi \int_a^b x f(x) dx$$

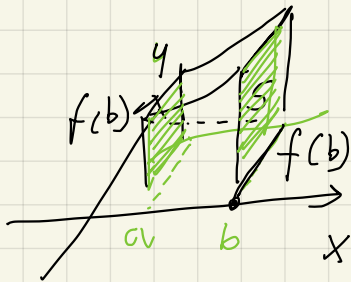
•)



METODO DELLE SEZIONI I

$$V = \int_a^b S(x) dx$$

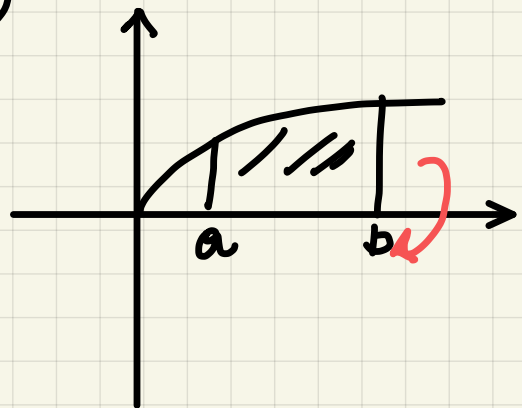
$S(x)$



$$S(x) = [f(x)]^2$$

•) $y = \sqrt{\frac{x+1}{x-2}}$ (3.4)

$$V = \pi \int_3^4 \frac{x+1}{x-2} dx$$

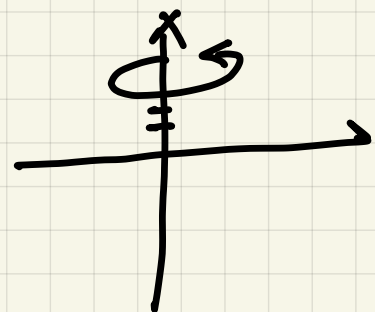


$$= \pi \int_3^4 \frac{x-2+3}{x-2} dx =$$

$$= \pi \int_3^4 \frac{\cancel{x-2} + 3}{\cancel{x-2}} dx + \pi \int_3^4 \frac{3}{x-2} = \pi \left[x + 3 \ln|x-2| \right]_3^4 =$$

$$= \pi \left[4 + 3 \ln 2 - 3 + 3 \cancel{\ln 1} \right] = \pi (1 + 3 \ln 2)$$

•) $y = \frac{2}{x} + 1$ [2;3] lungo y



$$\int_{-1}^1 f(y) dy$$

$$\frac{xy}{x} = \frac{2+x}{x} \rightarrow x(y-1) = 2$$

$$x = \frac{2}{y-1}$$

$$f(y) = \frac{2}{y-1} \quad I = \pi \int_2^3 \left(\frac{2}{y-1} \right)^2 dy$$

$$V = \pi \int_a^b [f(y)]^2 dy$$

$$V = \pi \int_2^3 \frac{4}{(y-1)^2} dy = 4\pi \int_2^3 (y-1)^{-2} dy =$$

$$= 4\pi \left[(-1) \cdot (y-1)^{-1} \right]_2^3 = 4\pi \left[-\frac{1}{y-1} \right]_2^3 = 4\pi \left[-\frac{1}{2} + \frac{1}{1} \right] =$$

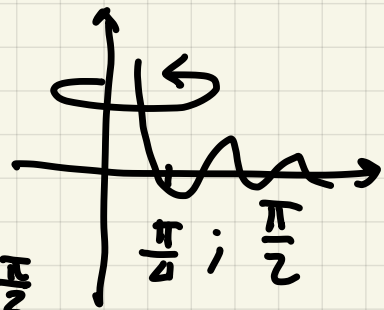
$$4\pi \cdot \frac{1}{2} = 2\pi$$

•) $y = \frac{3 \cos x}{x}$ assey $\rightarrow \frac{\pi}{4}; \frac{\pi}{2}$

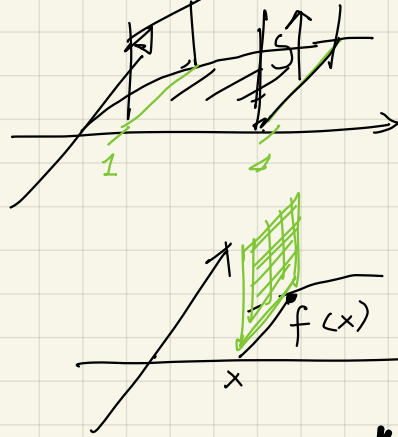
$$V = \pi \int x f(x) dx$$

$$V = \pi \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{3 \cos x}{x} dx = \pi \cdot 3 \left[\sin x \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} =$$

$$= 3\pi \left[\sin \frac{\pi}{2} - \sin \frac{\pi}{4} \right] = 3\pi \left(1 - \frac{\sqrt{2}}{2} \right) = \frac{6-3\sqrt{2}}{2} \pi.$$



$$\bullet) y = 2\sqrt{x}$$



(1;4)
quadrati.

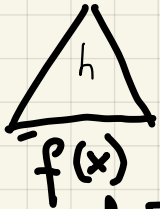
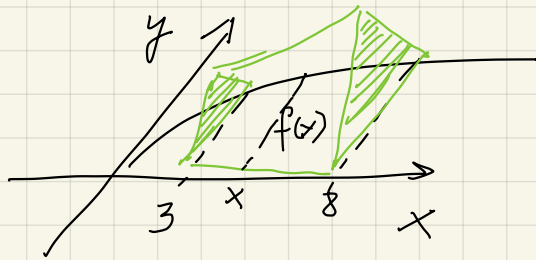
$$V = \int s(x) dx$$

$$s(x) = [f(x)]^2$$

$$V = \int_a^b 4 \cdot x dx = \int_1^4 4x dx = 4 \left[\frac{1}{2} x^2 \right]_1^4 = 4 \left(\frac{16}{2} - \frac{1}{2} \right) =$$

$$= 4 \cdot \frac{15}{2} = 30.$$

$$\bullet) y = 4\sqrt{x+1} \quad [3,8] \quad \text{triangoli equilateri}$$



$$h = \frac{f\sqrt{3}}{2} \rightarrow \frac{4\sqrt{x+1} \sqrt{3}}{2} \Rightarrow A(x) = \frac{b \times h}{2}$$

$$A(x) = 4\sqrt{x+1} \cdot \frac{\sqrt{3} \sqrt{x+1}}{2} = 4\sqrt{3} (x+1)$$

$$\rightarrow V = \int_3^8 4\sqrt{3} (x+1) dx = 4\sqrt{3} \left[\frac{1}{2} (x+1)^2 \right]_3^8 =$$

$$= 4\sqrt{3} \left(\frac{81}{2} - \frac{16}{2} \right) = 30\sqrt{3}.$$