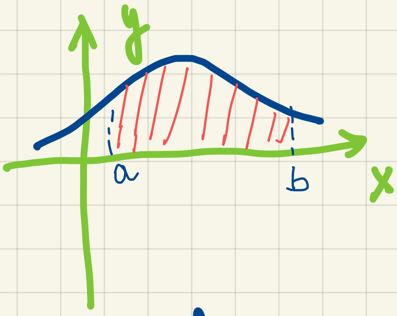
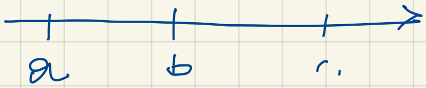
AREE FRA LE FUNZIONI



$$\int_a^b f(x)\,dx$$

REGOLA

$$\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$$



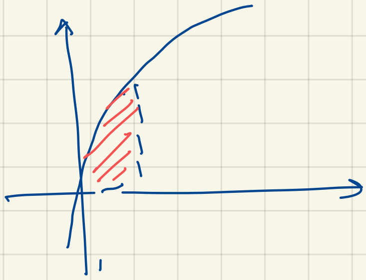
ES. $\ln(x+1)$ $[0,1]$

$$\int_0^1 \underset{u'}{1} \underset{v}{\ln(x+1)}\,dx$$

$u'=1$; $u=x$

$v=\ln(x+1)$

$v'=\frac{1}{x+1}$



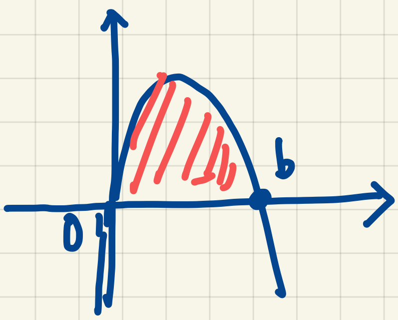
$$x\ln(x+1) - \underbrace{\int_0^1 \frac{x}{x+1}\,dx}_{I} \Rightarrow \int_0^1 \frac{x+1-1}{x+1}\,dx$$

$$I = \int_0^1 \frac{\cancel{x+1}}{\cancel{x+1}}\,dx - \int_0^1 \frac{1}{x+1}\,dx = [x]_0^1 - [\ln|x+1|]_0^1 =$$

$$= 1 - (\ln|2| - \ln(0+1)) = 1 - \ln 2 + 0 = I$$

$$= 1\ln(2) - \cancel{0\ln 1} - [1-\ln 2] = 2\ln 2 - 1$$

ES



$y = 6x - x^2$

$$\int_0^{?b} f(x)\,dx$$

$$\begin{cases} y = 6x - x^2 \\ y = 0 \end{cases}$$

$6x - x^2 = 0$

$x_1 = 0$

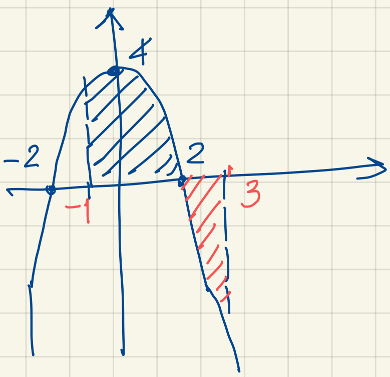
$6 - x = 0$

$x = 6$

$$\int_0^6 (6x - x^2)\,dx = \left[\frac{6}{2}x^2 - \frac{1}{3}x^3\right]_0^6 = 3\cdot 36 - 72 = \boxed{36}$$

ES $y = -x^2 + 4$ $[-1;3]$

$$\int_{-1}^2 (-x^2+4)\,dx - \int_2^3 (-x^2+4)\,dx$$

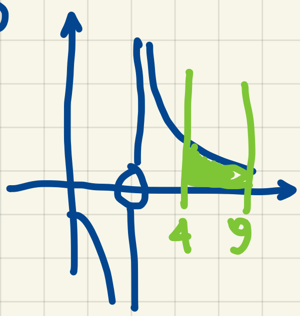


$$\int \to \left(-\frac{1}{3}x^3+4x\right)_{-1}^{2} \quad {}_{2}^{3}$$

$$\left[-\frac{1}{3}2^3+4\cdot 2-\left(-\frac{1}{3}(-1)^3+4(-1)\right)\right]-\left[-\frac{1}{3}(3)^3+4\cdot 3-\left(-\frac{1}{3}(2^3)+4\cdot 2\right)\right]$$

$$=\boxed{\frac{16}{3}}$$

ESERCIZIO



Calcolare l'area della regione di piano individuata dal grafico della funzione $f(x)=\dfrac{1}{\sqrt{x}-1}$, dalle rette $x=4$ e $x=9$ e dall'asse delle x.

(*Politecnico di Torino, Test di Analisi I*)

$[2+\ln 4]$

$$\int_4^9 \frac{1}{\sqrt{x}-1}\,dx \qquad \text{sost } \sqrt{x}-1=t \qquad \sqrt{x}=t+1 \qquad x=(t+1)^2=t^2+2t+1$$

$$dx=(2t+2)\,dt$$

$$\int \frac{1}{t}(2t+2)\,dt = 2\left[\int dt+\int\frac{1}{t}\,dt\right]_4^9=\left[2t+2\ln|t|\right]_4^9=$$

$$=\left[2(\sqrt{x}-1)+2\ln|\sqrt{x}-1|\right]_4^9=2(3-1)+2\ln(3-1)-2(2-1)$$

$$-2\ln(2-1)=4+2\ln 2-2-2\ln 1=\boxed{2+\ln 4}$$

ESERCIZIO

$$\text{tg } 0: \quad y-0=f'(0)(x-0)$$

$$f'(x)=-x+2 \qquad f'(0)=2$$

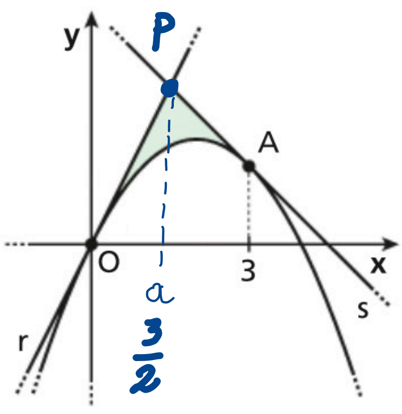
$$y=2x \qquad r$$

Le rette r e s sono tangenti alla parabola di equazione

$$y=-\frac{1}{2}x^2+2x$$

rispettivamente in O e A. Trova l'area della zona colorata.

$\left[\frac{9}{8}\right]$



$$\text{tg } 3: \quad y-\left(-\frac{9}{2}+6\right)=(-3+2)\cdot(x-3)$$

$$y=-x+\frac{9}{2} \qquad s$$

$$s \cap r \quad \begin{cases} y = 2x \\ y = -x + \frac{9}{2} \end{cases} \rightarrow \quad x = \frac{3}{2} \qquad A = \int_a^b UP - DWN$$

$$\int_0^{\frac{3}{2}} \left(\cancel{2x} + \frac{1}{2}x^2 - \cancel{2x}\right) dx + \int_{\frac{3}{2}}^{3} \left[\left(-x + \frac{9}{2}\right) - \left(-\frac{1}{2}x^2 + 2x\right)\right] dx$$

$$= \int_0^{\frac{3}{2}} \frac{1}{2}x^2 dx + \int_{\frac{3}{2}}^{3} \left(\frac{1}{2}x^2 - 3x + \frac{9}{2}\right) dx = \left[\frac{1}{6}x^3\right]_0^{\frac{3}{2}} +$$

$$+ \left[\frac{1}{6}x^3 - \frac{3}{2}x^2 + \frac{9}{2}x\right]_{\frac{3}{2}}^{3} = \cancel{\frac{1}{6}\cdot\frac{27}{8}} + \frac{27}{6} - \cancel{\frac{27}{2}} + \cancel{\frac{27}{2}} - \cancel{\frac{1}{6}\cdot\frac{27}{8}} + \frac{27}{8} - \frac{27}{4} = \boxed{\frac{9}{8}}$$