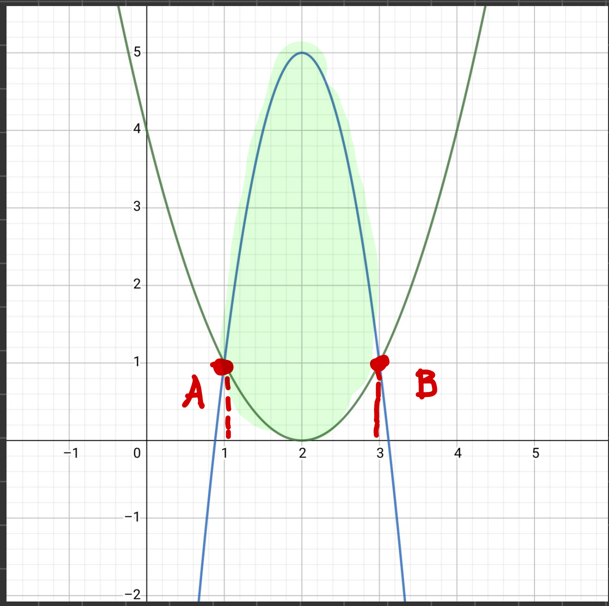
M5036 - Area compresa fra due due funzioni

5BLS 2020

$$\begin{cases} y = x^2 - 4x + 4 \\ y = -4x^2 + 16x - 11 \end{cases}$$



$$x^2 - 4x + 4 = -4x^2 + 16x - 11$$

$$5x^2 - 20x + 15 = 0$$

$$x^2 - 4x + 3 = 0$$

$$\Delta = 16 - 12 = 4 \qquad x_{1,2} = \frac{4 \pm 2}{2} \left\langle \begin{matrix} 3 \\ 1 \end{matrix} \right.$$

$A(1; \ldots) \qquad B(3; \ldots)$

$$\int_1^3 \left[\left(-4x^2 + 16x - 11 \right) - \left(x^2 - 4x + 4 \right) \right] dx$$

$$= \int_1^3 \left(-5x^2 + 20x - 15 \right) dx = \left[-\frac{5}{3} x^3 + \frac{20}{2} x^2 - 15x \right]_1^3$$

$$= \left[\left(-\frac{5}{3} 27 + 10 \cdot 9 - 15 \cdot 3 \right) - \left(-\frac{5}{3} + 10 - 15 \right) \right] =$$

$$= -\cancel{45} + \cancel{90} - \cancel{45} + \frac{5}{3} - 10 + 15 =$$

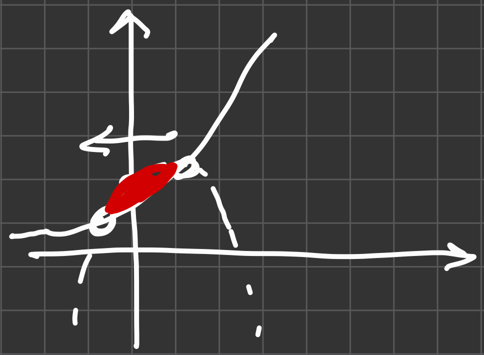
$$= \frac{5 - 30 + 45}{3} = \boxed{\frac{20}{3}}$$

ESERCIZIO

Dopo aver verificato che la parabola di equazione $y = -\frac{1}{2}x^2 + 1$ incontra la curva di equazione $y = 2^x$ nei punti $A\left(-1; \frac{1}{2}\right)$ e $B(0; 1)$, determina l'area della regione finita di piano delimitata dalle due curve. $\left[\frac{5}{6} - \frac{1}{\ln 4}\right]$

$$\begin{cases} y = -\frac{1}{2}x^2 + 1 \\ y = 2^x \end{cases}$$

$$2^x = -\frac{1}{2}x^2 + 1$$



A) $\begin{cases} \frac{1}{2} = -\frac{1}{2}(-1)^2 + 1 \\ \frac{1}{2} = -\frac{1}{2} + 1 \rightarrow \frac{1}{2} = \frac{1}{2} \end{cases}$

$\frac{1}{2} = 2^{(-1)} \rightarrow \frac{1}{2} = \frac{1}{2}$

B) $1 = -\frac{1}{2}0^2 + 1 \qquad 1 = 1$

$1 = 2^0 \qquad 1 = 1$

CHE A e B $\in$ alle 2 funzioni

$$\int_{-1}^{0} \left[\left(-\frac{1}{2}x^2 + 1\right) - \left(2^x\right)\right] dx =$$

$$= \left[\frac{1}{3}\left(-\frac{1}{2}\right)x^3 + x - \frac{2^x}{\ln 2}\right]_{-1}^{0} = \left(-\frac{1}{6}(0)^3 + 0 - \frac{2^0}{\ln 2}\right) -$$

$$- \left(-\frac{1}{6}(-1)^3 + (-1) - \frac{2^{-1}}{\ln 2}\right) =$$

$$= -\frac{1}{\ln 2} - \frac{1}{6} + 1 + \frac{1}{2\ln 2} = \frac{5}{6} + \frac{-2+1}{2\ln 2} =$$

$$= \frac{5}{6} - \frac{1}{2\ln 2} = \boxed{\frac{5}{6} - \frac{1}{\ln 4}}$$