Integrali di funzioni razionali fratte

1) GRADO N $\geq$ GRADO D

$$\int \frac{x^3+2x^2+x+1}{x^2+1}\,dx$$

DIVISIONE

$\frac{6}{2} = 3 \longrightarrow 3 \cdot 2 = 6$

$\frac{7}{2} = 3 \quad R = 1 \qquad 3 \cdot 2 + 1 = 7$

$$\begin{array}{l|l}
x^3 + 2x^2 + x + 1 & x^2 + 1 \\
\hline
-x^3 \qquad\quad - x & x + 2 \\
\hline
+ \quad // \quad 2x^2 + 0 + 1 & \\
\qquad -2x^2 \qquad -2 & \\
\hline
\qquad // \qquad\quad -1 &
\end{array}$$

$Q(x+2) \qquad R = -1$

$(x^2+1)(x+2) - 1 = x^3+2x^2+x+1$

$$\int \frac{\cancel{(x^2+1)}(x+2) - 1}{\cancel{x^2+1}}\,dx = \int (x+2)\,dx - \int \frac{1}{1+x^2}\,dx$$

$$\boxed{\frac{1}{2}x^2 + 2x - \operatorname{arctg} x + C}$$

2) GRADO DI N < DEL GRADO DI D

$$\int \frac{2x+6}{x^2+6x+8}\,dx = \ln|x^2+6x+8| + C$$

$$\frac{1}{5}\int \frac{7 \cdot 5}{5x-1}\,dx = \frac{7}{5}\ln|5x-1| + C$$

SOPRA

$$\frac{f'}{f}$$

SOTTO

2A) $\Delta > 0$

$$\int \frac{5x-1}{x^2-x-2}\,dx \qquad \Bigg\| \qquad x_1 \text{ e } x_2 \text{ ?}$$

$$x_1 = 2 \quad \vee \quad x_2 = -1$$

$$\boxed{\frac{A}{(x-x_1)} + \frac{B}{(x-x_2)}} \longrightarrow \frac{A}{(x-2)} + \frac{B}{(x+1)}$$

$$\frac{A(x+1) + B(x-2)}{(x-2)(x+1)} = \frac{Ax + A + Bx - 2B}{(x-2)(x+1)}$$

$$x^2 + x - 2x - 2$$

$$x^2 - x - 2$$

$$\frac{x(A+B) + (A-2B)}{(x-2)(x+1)} \qquad \begin{cases} A+B = 5 \\ A - 2B = -1 \end{cases}$$

$$A = +3 \qquad B = 2$$

$$(3+2)x + (3-4) = 5x - 1$$

$$\int \frac{5x-1}{x^2-x-2}\,dx \longrightarrow \int \left(\frac{3}{x-2} + \frac{2}{x+1}\right) dx$$

$$\int \frac{3}{x-2}\,dx + \int \frac{2}{x+1} + C = 3\ln|x-2| + 2\ln|x+1| + C$$

2B) $\displaystyle\int \frac{5+x}{x^2+6x+9}\,dx \rightarrow \Delta = 0$ denominatore

$$x^2+6x+9 \rightarrow (x+3)^2$$

$$\boxed{\frac{5+x}{x^2+6x+9} = \frac{A}{(x+3)} + \frac{B}{(x+3)^2}} \qquad \frac{A(x+3)+B}{(x+3)^2}$$

$$\frac{Ax+3A+B}{x^2+6x+9} \qquad \begin{cases} A = 1 \\ 3A+B = 5 \end{cases} = \begin{cases} A = 1 \\ 3\cdot 1 + B = 5 \end{cases}$$

$$\begin{cases} A = 1 \\ B = 2 \end{cases} \rightarrow \frac{1}{x+3} + \frac{2}{(x+3)^2} = \frac{5+x}{x^2+6x+9}$$

$$\int \left[\frac{1}{x+3} + \frac{2}{(x+3)^2}\right] dx = \int \frac{1}{x+3}\,dx + 2\int \frac{1}{(x+3)^2}\,dx$$

$$= \ln|x+3| + 2\int (x+3)^{-2}\,dx$$

$$\int f'(x)\,[f(x)]^n\,dx = \frac{1}{n+1}[f(x)]^{n+1} + C \qquad n=-2$$

$$= \ln|x+2| + 2\cdot\frac{1}{-2+1}(x+3)^{-2+1} + C =$$

$$\boxed{= \ln|x+2| - 2\frac{1}{(x+3)} + C}$$

▷ $\displaystyle\int \frac{x}{x^2-4x+4}\,dx$ $\quad \left[\ln|x-2| - \frac{2}{x-2} + c\right]$

▷ $\displaystyle\int \frac{x^2-x+1}{x^2-2x+1}\,dx$ $\quad \left[x + \ln|x-1| - \frac{1}{x-1} + c\right]$

▷ $\int \frac{2x-1}{x^2+2x+1}\,dx$ $\left[\ln(x+1)^2+\frac{3}{x+1}+c\right]$

PROVARE A RIPETERE L'INTEGRALE SVOLTO E RISO[illegible] I TRE ESERCIZI.

CASO 2C) $\int \frac{1}{x^2+4x+5}\,dx$ $\Delta<0$

$\Delta = 16-4\cdot 5=-4$

METODO DI COMPLETAMENTO DEL QUADRATO

$x^2+4x+4+1 \rightarrow (x^2+4x+4)+1 =$

$\hookrightarrow (x+2)^2$

$(x+2)^2+1$

$\int \frac{1}{1+x^2}\,dx = \text{arctg}\,x + C$

$\int \frac{1}{(x+2)^2+1}\,dx$

$x+2=t \rightarrow x=t-2$

$dx=dt$

$\int \frac{1}{t^2+1}\,dt \rightarrow \text{arctg}\,t + C \rightarrow \boxed{\text{arctg}(x+2)+C}$

▷ $\int \frac{x-1}{x^2-4x+5}\,dx = \frac{1}{2}\int \frac{2(x-1)}{x^2-4x+5}\,dx$

$\frac{1}{2}\int \frac{2x-2}{x^2-4x+5}\,dx = \frac{1}{2}\int \frac{2x-2-2+2}{x^2-4x+5}\,dx$

$$= \frac{1}{2}\int \frac{2x-4}{x^2-4x+5}\,dx + \frac{1}{\cancel{2}}\int \frac{+\cancel{2}}{x^2-4x+5}\,dx$$

$$= \underbrace{\frac{1}{2}\ln|x^2-4x+5|}_{\text{CONGELATO}} + \underbrace{\int \frac{1}{x^2-4x+4+1}\,dx}_{I_1}$$

$$I_1 = \int \frac{1}{(x-2)^2+1}\,dx$$

$t = x-2$

$x = t+2$

$dx = dt$

$$= \int \frac{1}{t^2+1}\,dt = \operatorname{arctg} t + C =$$

$$= \operatorname{arctg}(x-2) + C$$

$$\boxed{\frac{1}{2}\ln|x^2-4x+5| + \operatorname{arctg}(x-2) + C}$$