DERIVATE PARTICOLARI

M5018

POTENZA

$$y = f(x) = x^n$$
$$f'(x) = n\, x^{n-1}$$

RADICE

$$y = f(x) = \sqrt{x} \rightarrow x^{\frac{1}{2}} \rightarrow \frac{1}{2} x^{-\frac{1}{2}}$$
$$y' = f'(x) = \frac{1}{2\sqrt{x}}$$

COSTANTE

$$y = f(x) = k \rightarrow k\, x^0 \rightarrow 0 \cdot k^{0-1}$$
$$y' = f'(x) = 0$$

SENO

$$y = f(x) = \sin x$$

$$\lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h} = \lim_{h\to 0} \frac{\sin x \cos h + \sin h \cos x - \sin x}{h} =$$

$$= \lim_{h\to 0} \frac{\sin x\,[\cos h - 1] + \sin h \cos x}{h} =$$

$$= \lim_{h\to 0} \frac{\sin x\,[\cos h - 1]}{h} + \lim_{h\to 0} \frac{\sin h \cos x}{h}$$

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$0 \qquad\qquad 1$$

$$= \boxed{\cos x}$$

$$y' = f'(x) = \cos x$$

COSENO

$$y = f(x) = \cos x$$

$$\lim_{h\to 0} \frac{\cos(x+h) - \cos x}{h} = \lim_{h\to 0} \frac{\cos x \cos h - \operatorname{sen} x \operatorname{sen} h - \cos x}{h}$$

$$= \lim_{h\to 0} \frac{\cos x \left[\cos h - 1\right] - \operatorname{sen} x \operatorname{sen} h}{h} = \boxed{-\operatorname{sen} x}$$

$$y' = f'(x) = -\operatorname{sen} x$$

esponenziale

$$y = f(x) = e^x$$

$$\lim_{h\to 0} \frac{e^{x+h} - e^x}{h} = \lim_{h\to 0} \frac{e^x \cdot e^h - e^x}{h} =$$

$$\lim_{h\to 0} \frac{e^x\,(e^h-1)}{h} = e^x$$

$\to$ 1 (limite fondamentale)

$$y' = f'(x) = e^x$$

SE $y = a^x$ $(2^x, 3^x \ldots)$

$$y' = a^x \cdot \ln a$$

$$D\,2^x = 2^x \cdot \ln 2$$

LOGARITMO

$$y = f(x) = \log_a x$$

$$\lim_{h\to 0} \frac{\log_a(x+h) - \log_a x}{h} = \lim_{h\to 0} \frac{\log_a \frac{(x+h)}{x}}{h}$$

$$\lim_{h\to 0} \frac{\log_a\left(1+\frac{h}{x}\right)}{h} = \lim_{h\to 0} \frac{1}{h}\log_a\left(1+\frac{h}{x}\right) =$$

$$\lim_{h\to 0} \log_a\left(1+\frac{h}{x}\right)^{\frac{1}{h}} =$$

$a \log b = \log b^a$ 3^a propr.

$$\log_a \lim_{h\to 0}\left(1+\frac{h}{x}\right)^{\frac{1}{h}} =$$

$\frac{h}{x} = \frac{1}{t}$ $h \to 0$

$t = \frac{x}{h}$ $t \to \infty$

$$\log_a \lim_{t\to 0}\left(1+\frac{1}{t}\right)^{\frac{t}{x}} =$$

$h = \frac{x}{t}$ $\frac{1}{h} = \frac{t}{x}$

$$= \log_a \lim_{t\to 0}\left[\left(1+\frac{1}{t}\right)^{t}\right]^{\frac{1}{x}} = \log_a e^{\frac{1}{x}} = \boxed{\frac{1}{x}\log_a e}$$

IN PARTICOLARE SE $a = e$

$$\frac{1}{x} \cdot \log_e e = \frac{1}{x} \ln e = \frac{1}{x}$$

$$y = \ln x \rightarrow \boxed{y' = \frac{1}{x}}$$