

# DERIVATE PARTICOLARI

M5018



## POTENZA

$$y = f(x) = x^n$$
$$f'(x) = n x^{n-1}$$

## RADICE

$$y = f(x) = \sqrt{x} \rightarrow x^{\frac{1}{2}} \rightarrow \frac{1}{2} x^{-\frac{1}{2}}$$
$$y' = f'(x) = \frac{1}{2\sqrt{x}}$$

## COSTANTE

$$y = f(x) = k \rightarrow k x^0 \rightarrow 0 \cdot k^{0-1}$$
$$y' = f'(x) = 0$$

# SENO

$$y = f(x) = \sin x$$

$$\lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h} = \lim_{h \rightarrow 0} \frac{\sin x \cosh + \sin h \cos x - \sin x}{h} =$$

$$= \lim_{h \rightarrow 0} \frac{\sin x [\cosh - 1] + \sin h \cos x}{h} =$$

$$= \lim_{h \rightarrow 0} \frac{\sin x [\underbrace{\cosh - 1}_h]}{h} + \lim_{h \rightarrow 0} \frac{\sin h \underbrace{\cos x}_h}{h}$$

Diagrammatic annotations: In the first term, a green cloud highlights the fraction  $\frac{\cosh - 1}{h}$  with an arrow pointing to a '0' below it. In the second term, a green cloud highlights the fraction  $\frac{\sin h \cos x}{h}$  with an arrow pointing to a '1' below it.

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$= \boxed{\cos x}$$

$$y' = f'(x) = \cos x$$

## COSENO

$$y = f(x) = \cos x$$

$$\lim_{h \rightarrow 0} \frac{\cos(x+h) - \cos x}{h} = \lim_{h \rightarrow 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cos x [\cos h - 1] - \sin x \sin h}{h} = \boxed{-\sin x}$$

$$y' = f'(x) = -\sin x$$

## esponenziale

$$y = f(x) = e^x$$

$$\lim_{h \rightarrow 0} \frac{e^{x+h} - e^x}{h} = \lim_{h \rightarrow 0} \frac{e^x \cdot e^h - e^x}{h} =$$

$\lim_{h \rightarrow 0}$

$$\frac{e^x (e^h - 1)}{h} = e^x$$

$\rightarrow 1$  (limite fondamentale)

$$y' = f'(x) = e^x$$

SE  $y = a^x$  ( $2^x, 3^x, \dots$ )

$$y' = a^x \cdot \ln a$$

$$D 2^x = 2^x \cdot \ln 2$$

## LOGARITMO

$$y = f(x) = \log_a x$$

$\lim_{h \rightarrow 0}$

$$\frac{\log_a (x+h) - \log_a x}{h}$$

$= \lim_{h \rightarrow 0}$

$$\frac{\log_a \frac{(x+h)}{x}}{h}$$

$$\lim_{h \rightarrow 0} \frac{\log_a \left( 1 + \frac{h}{x} \right)}{h} = \lim_{h \rightarrow 0} \frac{1}{h} \log_a \left( 1 + \frac{h}{x} \right) =$$

$$\lim_{h \rightarrow 0} \log_a \left( 1 + \frac{h}{x} \right)^{\frac{1}{h}} =$$

$$a \log b = \log b^a \quad 3^{\circ} \text{ propr.}$$

$$\log_a \lim_{h \rightarrow 0} \left( 1 + \frac{h}{x} \right)^{\frac{1}{h}} =$$

$$\frac{h}{x} = \frac{1}{t} \quad h \rightarrow 0$$

$$t = \frac{1}{x} \quad t \rightarrow \infty$$

$$\log_a \lim_{t \rightarrow \infty} \left( 1 + \frac{1}{t} \right)^{\frac{t}{x}} =$$

$$h = \frac{1}{t} \quad \frac{1}{h} = t$$

$$= \log_a \lim_{t \rightarrow \infty} \left[ \left( 1 + \frac{1}{t} \right)^t \right]^{\frac{1}{x}} = \log_a e^{\frac{1}{x}} =$$

$$\frac{1}{x} \log_a e$$

**IN PARTICOLARE SE  $a = e$**

$$\frac{1}{x} \cdot \log_e e = \frac{1}{x} \ln e = \frac{1}{x}$$

$$y = \ln x \rightarrow \boxed{y' = \frac{1}{x}}$$