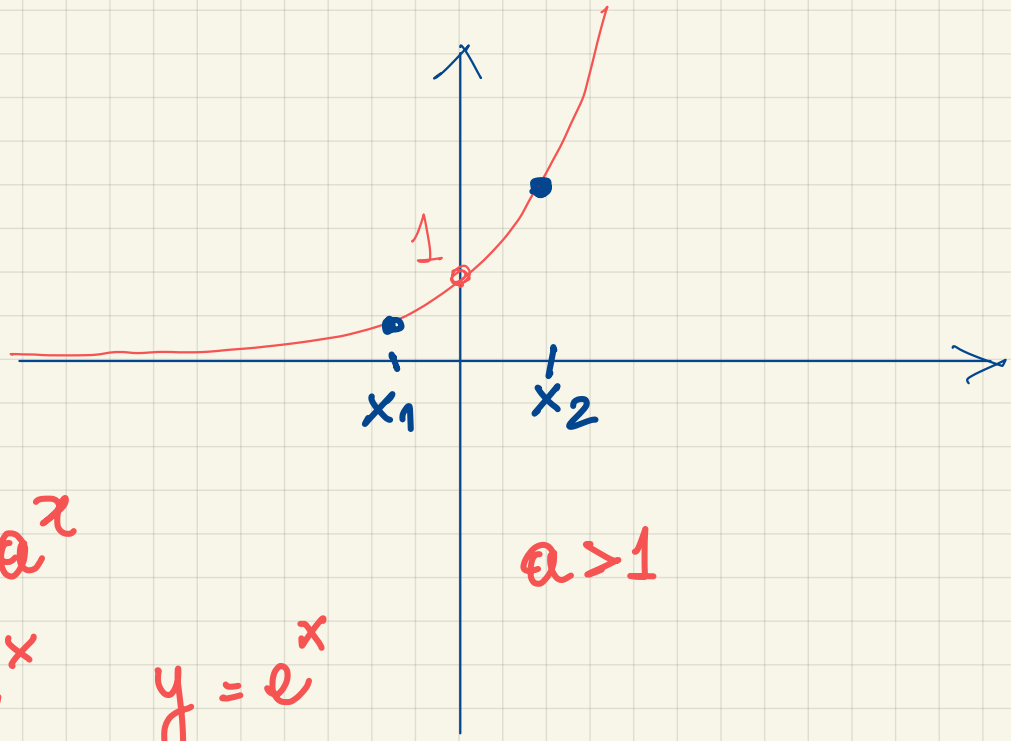
DISEQUAZIONI ESPONENZIALI

M3050

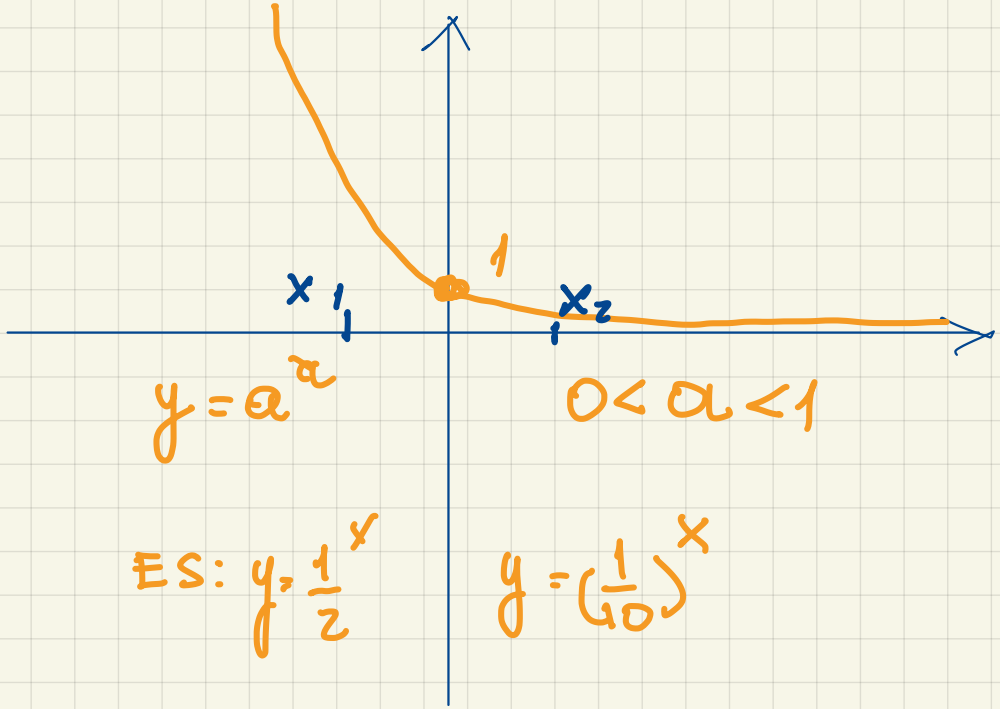
LA FUNZIONE $y = a^x$



$y = a^x$

$a > 1$

$y = 2^x$ $\quad y = e^x$



$y = a^x$

$0 < a < 1$

ES: $y = \frac{1}{2}^x$ $\quad y = (\frac{1}{10})^x$

e = numero di EULERO costante $e = 2,7\ldots$

1) $4^x \leq 32$ $\quad 2^{2x} \leq 2^5$ $\quad$ base >1

conservo il verso:

$2x \leq 5$ $\quad x \leq \frac{5}{2}$

2) $\left(\frac{3}{2}\right)^x < \frac{27}{8}$ $\quad \left(\frac{3}{2}\right)^x < \left(\frac{3}{2}\right)^3$ $\quad$ base $\frac{3}{2} > 1$

conservo il verso

$x < 3$

3) $\left(\frac{3}{2}\right)^x < \frac{8}{27}$ $\quad \left(\frac{3}{2}\right)^x < \left(\frac{3}{2}\right)^{-3}$ $\quad$ base >1 $\rightarrow$ $x < -3$

4) $3^{2x+2} < \frac{1}{3}$ $\quad 3^{2x+2} < 3^{-1}$ $\quad$ base >1 $\quad 2x+2 < -1 \rightarrow x < -\frac{3}{2}$

5) $\left(\frac{1}{4}\right)^{x-1} < 64 \rightarrow \left(\frac{1}{2}\right)^{2(x-1)} < \left(\frac{1}{2}\right)^{-6}$ $\quad$ base $0 < a < 1$ $\triangle$

NON CONSERVO $\quad 2(x-1) > -6 \quad 2x-2 > -6 \quad x > -2$

1) $4^x \leq 32$ $\quad \left[x \leq \frac{5}{2}\right]$

2) $\left(\frac{3}{2}\right)^x < \frac{27}{8}$ $\quad [x < 3]$

3) $\left(\frac{3}{2}\right)^x < \frac{8}{27}$ $\quad [x < -3]$

4) $3^{2x+2} < \frac{1}{3}$ $\quad \left[x < -\frac{3}{2}\right]$

5) $\left(\frac{1}{4}\right)^{x-1} < 64$ $\quad [x > -2]$

6) $0{,}1^x \leq 100$ $\quad [x \geq -2]$

6) $0{,}1^x \leq 100$ $\left(\frac{1}{10}\right)^x \leq 100$ $10^{-x} \leq 10^2$ $-x \leq 2$

$+x \geq -2$

7) $\frac{9 \cdot 3^{-x}}{9^x + 3^{2x}} > \frac{27}{2}$ $\left[x < -\frac{1}{3}\right]$

$3^x = t$

$\frac{9 \cdot 3^{-x}}{3^{2x} + 3^{2x}} > \frac{27}{2} \rightarrow \frac{9 \cdot t^{-1}}{t^2 + t^2} > \frac{27}{2}$

$\frac{9}{t} \cdot \frac{1}{2t^2} > \frac{27}{2}$ $\frac{9}{2t^3} > \frac{27}{2} \rightarrow \frac{9}{2t^3} > \frac{27t^3}{2t^3}$

N $\frac{9 - 27t^3}{2t^3} > 0$

D

N > 0

$9 - 27t^3 > 0$

$-27t^3 > -9$

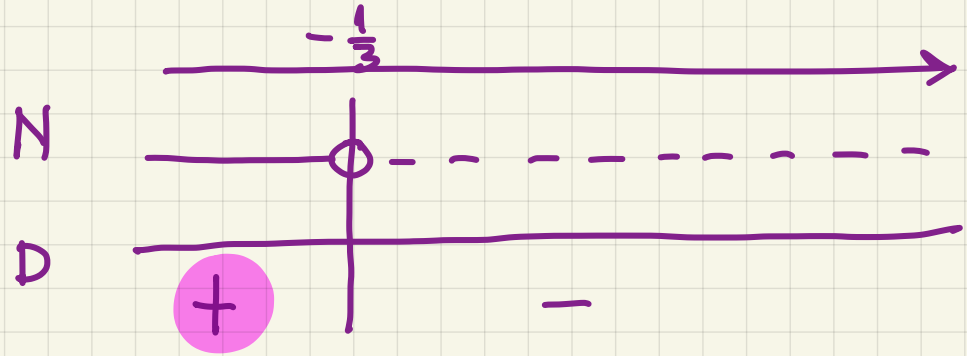
$27t^3 < 9$ $t^3 < \frac{9}{27}$

$t^3 < \frac{1}{3} \rightarrow (3^x)^3 < 3^{-1}$

$3^{3x} < 3^{-1}$ $3x < -1$ N $x < -\frac{1}{3}$

$D>0 \qquad 2t^3>0 \qquad t^3>0 \qquad 3^{3x}>0 \quad \forall\, x\in\mathbb{R}$

UN ESPONENZIALE É SEMPRE POSITIVO



$$x < -\frac{1}{3}$$

⊛ $\dfrac{9t^{-1}}{2t^2} > \dfrac{27}{2}$ come gestire $9t^{-1}$

$$\frac{9\cdot\frac{1}{t}}{2t^2} \Rightarrow \frac{\frac{9}{t}}{2t^2} \quad \text{(num: } \tfrac{9}{t}\text{, denom: } 2t^2\text{)}$$

$$\frac{9}{t}\cdot\frac{1}{2t^2} \to \frac{9}{2t^3}$$