

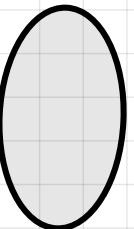
ELLISSE - TROVARE L'EQUAZIONE



M3041

1) fuochi su y $\rightarrow b > a$ passa per P(z; $\frac{5}{3}\sqrt{5}$)

$$e = \frac{4}{5}$$



sezione maggiore b

$$e = \frac{c}{b}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Pone per P

$$\frac{z^2}{a^2} + \frac{(\frac{5}{3}\sqrt{5})^2}{b^2} = 1 \rightarrow \frac{4}{a^2} + \frac{125}{9b^2} = 1$$

$$\frac{36b^2 + 125a^2}{3a^2b^2} = \frac{3a^2b^2}{3a^2b^2} \rightarrow \begin{cases} 125a^2 + 36b^2 = 9a^2b^2 \\ c = \frac{4}{5}b \end{cases}$$

eccentricità

$$c^2 = b^2 - a^2$$

$$\left\{ \begin{array}{l} 12s a^2 + 36 b^2 = 9 a^2 b^2 \\ c^2 = \frac{16}{25} b^2 \\ c^2 = b^2 - a^2 \end{array} \right.$$

$$\left\{ \begin{array}{l} \text{---} \\ b^2 - a^2 = \frac{16}{25} b^2 \\ \text{---} \end{array} \right.$$

$$\left\{ \begin{array}{l} \text{---} \\ b^2 - \frac{16}{25} b^2 = a^2 \\ \text{---} \end{array} \right.$$

$$\left\{ \begin{array}{l} \text{---} \\ \frac{9}{25} b^2 = a^2 \\ \text{---} \end{array} \right.$$

↑ sost.

$$\left\{ \begin{array}{l} 12s \frac{9}{25} b^2 + 36 b^2 = 9 \frac{9}{25} b^2 \cdot b^2 \\ \text{---} \\ \text{---} \end{array} \right.$$

$$\frac{81}{25} b^4 - \frac{112s}{25} b^2 - 36 b^2 = 0 \rightarrow \cancel{\frac{81}{25} b^4} - \cancel{\frac{202s}{25} b^2} = 0$$

$$b^2 (81 b^2 - 202s) = 0$$

$$\rightarrow b^2 = 0 \quad \text{NO}$$

$$\rightarrow 81 b^2 - 202s = 0 \rightarrow b^2 = \frac{202s}{81} = 25$$

$$b^2 = 25 \quad \text{e} \quad a^2 = \frac{9}{25} \cdot 25 = 9$$

$$c^2 = 25 - 9 = 16$$

RIASSUMO

$$a^2 = 9$$

$$b^2 = 25$$

$$c^2 = 16$$

$$\frac{x^2}{9} + \frac{y^2}{25} = 1$$

Trova l'equazione dell'ellisse avente due dei suoi vertici nei punti di intersezione della retta di equazione $x - 3y + 9 = 0$ con gli assi cartesiani.

$$\left[\frac{x^2}{81} + \frac{y^2}{9} = 1 \right]$$

$$V_1 \left\{ \begin{array}{l} y = \frac{1}{3}x + 3 \\ y = 0 \end{array} \right.$$

$$0 = \frac{1}{3}x + 3$$

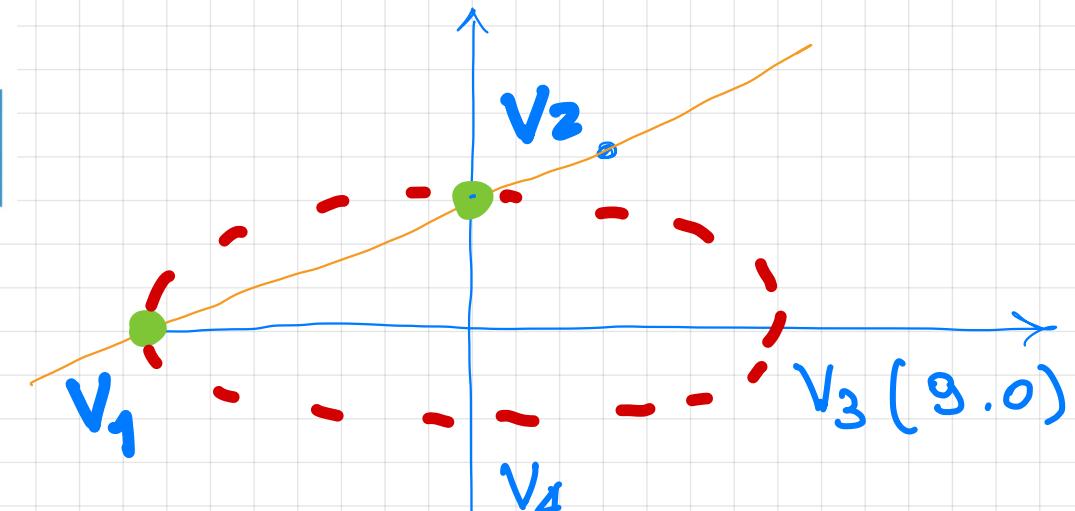
$$0 = x + 9 \rightarrow x = -9$$

$$V_1(-9; 0)$$

$$V_2 = (0; 3)$$

$$V_4(0; -3)$$

$$y = \frac{1}{3}x + 3 \quad \text{retta esp.}$$



$$a = 9 \quad b = 3$$

$$\frac{x^2}{81} + \frac{y^2}{9} = 1$$

Determina l'equazione della circonferenza che ha centro nell'origine e raggio uguale a $2\sqrt{5}$ e quella dell'ellisse riferita ai propri assi che ha un fuoco nel punto $F(3; 0)$ e che passa per il punto $P\left(2; \frac{4}{5}\sqrt{21}\right)$. Calcola l'area del quadrilatero formato dai punti di intersezione delle due curve.

$$\left[x^2 + y^2 = 20; 16x^2 + 25y^2 = 400; \frac{160}{9}\sqrt{5} \right]$$

- circonferenza

$$x^2 + y^2 = 20$$

- ellisse

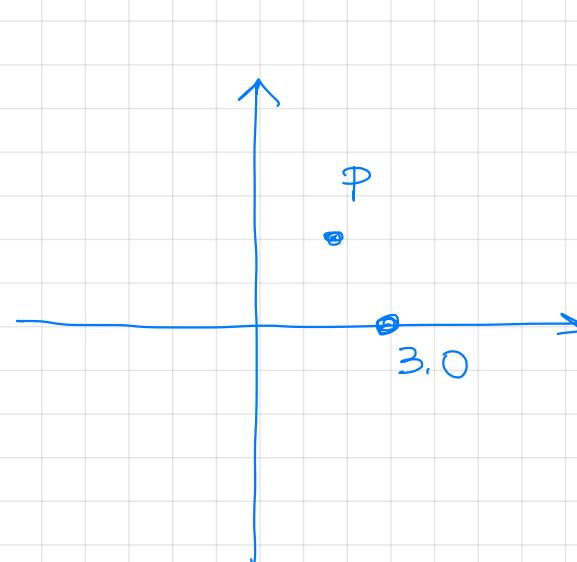
$$P\left(2; \frac{4}{5}\sqrt{21}\right) \quad F(3; 0)$$

$$c=3$$

$$a>b$$

$$\frac{4}{a^2} + \frac{336}{25b^2} = 1 \quad \text{per } P$$

$$\frac{100b^2 + 336a^2}{25a^2b^2} = \frac{25a^2b^2}{25a^2b^2}$$



$$\begin{cases} 100b^2 + 336a^2 = 25a^2b^2 \\ g = a^2 - b^2 \end{cases}$$

$$\begin{cases} 100(a^2-g) + 336a^2 = 25a^2(a^2-g) \\ b^2 = a^2 - g \end{cases}$$

$$100a^2 - 900 + 336a^2 = 25a^4 - 225a^2$$

$$25a^4 - 661a^2 + 900 = 0$$

$$25y^2 - 661y + 900 = 0$$

$$y_{1,2} = \frac{661 \pm \sqrt{346821}}{50}$$

$$\frac{661 + 589}{50} = 25$$

$$\frac{661 - 589}{50} = \frac{36}{25}$$

25) SE $c^2 = a^2 - b^2$ $g = 25 - b^2 \rightarrow b^2 = 16$

36) $0 = \frac{36}{25} - b^2$

$$b^2 = \frac{36}{25} - 9 = \frac{36 - 225}{25} = -\frac{189}{25}$$

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

$$\begin{cases} x^2 + y^2 = 20 \\ \frac{x^2}{25} + \frac{y^2}{16} = 1 \end{cases}$$

$$\begin{cases} x^2 + y^2 = 20 \\ 16x^2 + 25y^2 = 400 \end{cases}$$

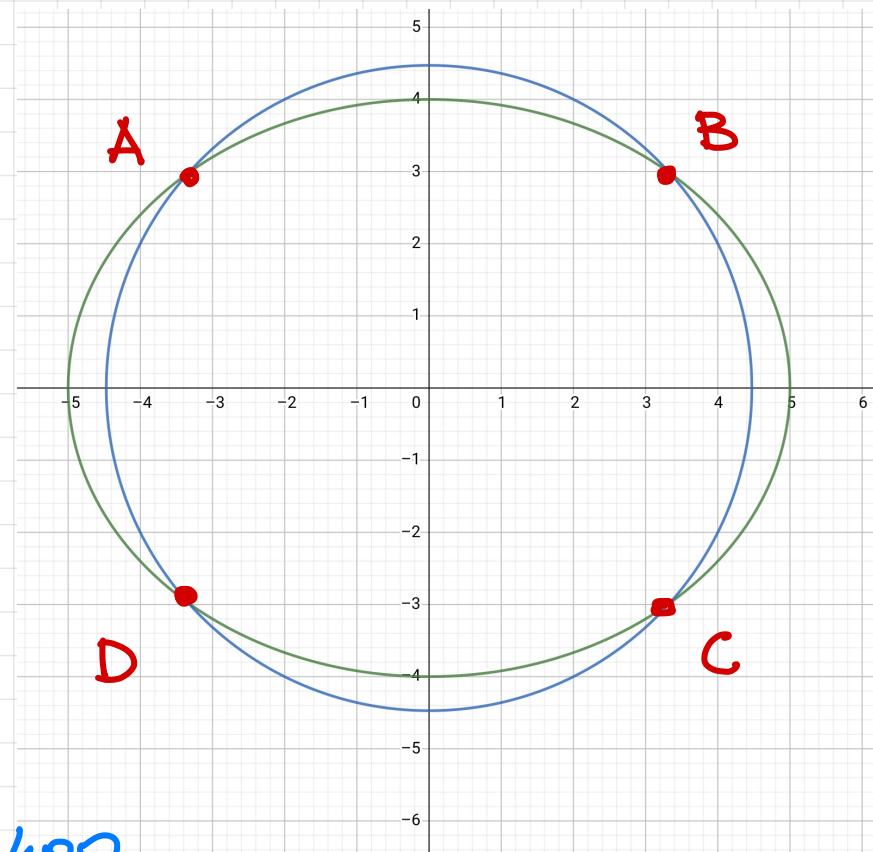
$$\begin{cases} \text{---} \\ 320 - 16y^2 + 25y^2 = 400 = \begin{cases} \text{---} \\ 9y^2 = 80 \end{cases} \end{cases}$$

$$\begin{cases} x^2 = \frac{100}{9} \\ \text{---} \end{cases}$$

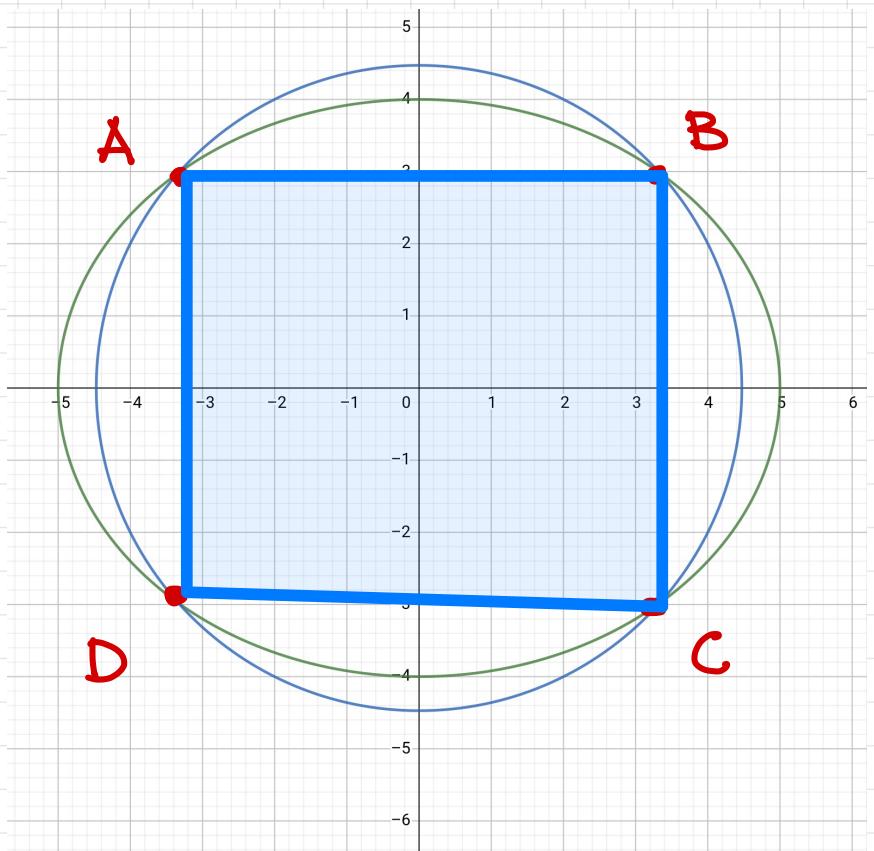
$$x_1 = \frac{10}{3} \quad x_2 = -\frac{10}{3}$$

$$\begin{cases} x^2 = 20 - \frac{80}{9} \\ y^2 = \frac{80}{9} \end{cases}$$

$$\begin{aligned} y_1 &= \frac{4\sqrt{5}}{3} & y_2 &= -\frac{4\sqrt{5}}{3} \end{aligned}$$



$$A\left(-\frac{10}{3}; \frac{4\sqrt{5}}{3}\right) \quad B\left(\frac{10}{3}; \frac{4\sqrt{5}}{3}\right) \quad C\left(\frac{10}{3}; -\frac{4\sqrt{5}}{3}\right) \quad D\left(-\frac{10}{3}; -\frac{4\sqrt{5}}{3}\right)$$



AREA: $\overline{AB} \cdot \overline{BC}$

$$\overline{BC} = 2 \cdot \frac{4\sqrt{5}}{3}$$

$$\overline{AB} = 2 \cdot \frac{10}{3}$$

$$\text{Area} = \frac{8\sqrt{5}}{3} \cdot \frac{20}{3} = \boxed{\frac{160\sqrt{5}}{9}}$$

AREA DELL'ELLISSE

$$A = \pi ab$$