FORMULE GONIOMETRICHE
Parametriche, Werner e Prostaferesi

M4012

FORMULE PARAMETRICHE

$\operatorname{sen} 2\alpha = 2 \operatorname{sen}\alpha \cos\alpha \longrightarrow 2\alpha \Rightarrow \rightarrow \alpha = 2\,\frac{\alpha}{2}$

$\operatorname{sen}\alpha = 2 \operatorname{sen}\frac{\alpha}{2} \cos\frac{\alpha}{2}$

$$\operatorname{sen}\alpha = \frac{2 \operatorname{sen}\frac{\alpha}{2} \cos\frac{\alpha}{2}}{1} \longrightarrow \operatorname{sen}\alpha = \frac{2 \operatorname{sen}\frac{\alpha}{2} \cos\frac{\alpha}{2}}{\underbrace{\operatorname{sen}^2\frac{\alpha}{2} + \cos^2\frac{\alpha}{2}}_{1}}$$

$\div \cos^2\frac{\alpha}{2}$ SOPRA E SOTTO

$$\operatorname{sen}\alpha = \frac{\dfrac{2 \operatorname{sen}\frac{\alpha}{2} \cos\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}}{\dfrac{\operatorname{sen}^2\frac{\alpha}{2} + \cos^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}}$$

POSSO DIVIDERE SENZA PROBLEMI

SÍ, se $\cos^2\frac{\alpha}{2} \neq 0$

$$\operatorname{sen}\alpha = \frac{\dfrac{2\operatorname{sen}\frac{\alpha}{2}\,\cancel{\cos\frac{\alpha}{2}}}{\cos^{\cancel{2}}\frac{\alpha}{2}}}{\dfrac{\operatorname{sen}^2\frac{\alpha}{2}+\cos^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}}$$

SEMPLIFICO E SPEZZO IL DENOMINATORE

$$\operatorname{sen}\alpha = \frac{2\operatorname{tg}\frac{\alpha}{2}}{\dfrac{\operatorname{sen}^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}+\dfrac{\cancel{\cos^2\frac{\alpha}{2}}}{\cancel{\cos^2\frac{\alpha}{2}}}}$$

$$\rightarrow \quad \frac{a+b}{c} = \frac{a}{c}+\frac{b}{c}$$

$$\operatorname{sen}\alpha = \frac{2\,\operatorname{tg}\frac{\alpha}{2}}{\operatorname{tg}^2\frac{\alpha}{2}+1}$$

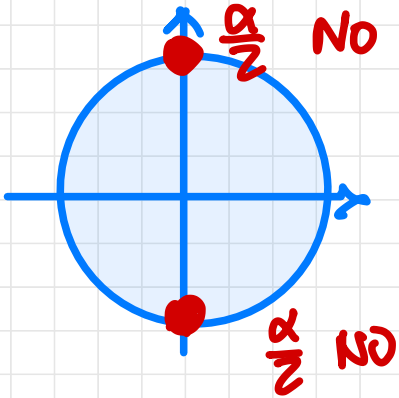
VERO SE $\cos^2\frac{\alpha}{2} \neq 0$

CIOÉ $\cos\frac{\alpha}{2} \neq 0$

A PATTO CHE $\frac{\alpha}{2} \neq \frac{\pi}{2} + k\pi \quad \times 2$

CIOÉ $\boxed{\alpha \neq \pi + 2k\pi}$

CONDIZIONE PER POTER APPLICARE LE F. PARAMETRICHE



$$\operatorname{sen}\alpha = \frac{2\,\operatorname{tg}\frac{\alpha}{2}}{\operatorname{tg}^2\frac{\alpha}{2}+1} \quad ; \quad \cos\alpha = \frac{1-\operatorname{tg}^2\alpha}{1+\operatorname{tg}^2\alpha}$$

pongo $t = \operatorname{tg}\frac{\alpha}{2}$

$$\boxed{\operatorname{sen}\alpha = \frac{2t}{1+t^2} \quad ; \quad \cos\alpha = \frac{1-t^2}{1+t^2}}$$

FORMULE PARAMETRICHE

FORMULE DI PROSTAFERESI E WERNER

SOMMA DI SENI
SOTTRAZIONE

$$\sin p + \sin q = 2 \sin \frac{p+q}{2} \cos \frac{p-q}{2};$$

$$\sin p - \sin q = 2 \cos \frac{p+q}{2} \sin \frac{p-q}{2};$$

SOMMA DI COSENI
SOTTRAZIONE

$$\cos p + \cos q = 2 \cos \frac{p+q}{2} \cos \frac{p-q}{2};$$

$$\cos p - \cos q = -2 \sin \frac{p+q}{2} \sin \frac{p-q}{2}.$$

PRODOTTI :

$$\sin \alpha \cos \beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)];$$

$$\cos \alpha \cos \beta = \frac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)];$$

$$\sin \alpha \sin \beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)].$$

ES: $\operatorname{sen} \underset{p}{\frac{\pi}{4}} - \operatorname{sen} \underset{q}{\frac{\pi}{12}} = 2 \cos \frac{\overset{p}{\frac{\pi}{4}} + \overset{q}{\frac{\pi}{12}}}{2} \operatorname{sen} \frac{\overset{p}{\frac{\pi}{4}} - \overset{q}{\frac{\pi}{12}}}{2}$

$= 2 \cos \frac{\pi}{3} \operatorname{sen} \frac{\pi}{6} = 2 \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$

PROVIAMO AD APPLICARE

1) $\dfrac{\sin(60° + \alpha) + \sin(60° - \alpha)}{\cos 3\alpha + \cos 5\alpha}$ $\left[\dfrac{\sqrt{3}}{2\cos 4\alpha}\right]$

2) $\dfrac{\sin 5\alpha - \sin 3\alpha}{\cos 3\alpha - \cos 5\alpha}$ $[\cot 4\alpha]$

3) $\dfrac{\cos\alpha + \cos 3\alpha}{\sin 5\alpha + \sin 3\alpha}$ $\left[\dfrac{1}{2\sin 2\alpha}\right]$

1) $\dfrac{\text{SOMMA DI SENI}}{\text{SOMMA DI COSENI}} \rightarrow$ PROSTAF. $= 2\,\text{sen}\,\dfrac{p+q}{2}\cos\dfrac{p-q}{2}$

$$\frac{\text{sen}(\overset{p}{60° + \alpha}) + \text{sen}(\overset{q}{60° - \alpha})}{\cos \underset{p}{3\alpha} + \cos \underset{q}{5\alpha}} = \frac{2\,\text{sen}\,\dfrac{(60 + \cancel{\alpha} + 60 - \cancel{\alpha})}{2} \cdot \cos\dfrac{\cancel{60} + \alpha - \cancel{60} + \alpha}{2}}{2\cos\dfrac{3\alpha + 5\alpha}{2}\cos\dfrac{3\alpha - 5\alpha}{2}}$$

$$= \frac{\cancel{2}\sin 60^\circ \cos\alpha}{\cancel{2}\cos 4\alpha \cos(-\alpha)} = \frac{\sqrt{3}}{2}\,\frac{\cancel{\cos\alpha}}{\cos 4\alpha \cdot \cancel{\cos\alpha}}$$

$\cos(-\alpha) = \cos(\alpha)$

$$\frac{\sqrt{3}}{2\cos 4\alpha}$$

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2) $$\frac{\sin 5\alpha - \sin 3\alpha}{\cos 3\alpha - \cos 5\alpha} = \frac{2\cos\frac{8\alpha}{2}\,\sin\frac{2\alpha}{2}}{-2\sin\frac{8\alpha}{2}\,\sin\frac{-2\alpha}{2}}$$

$$= \frac{\cancel{2}\cos 4\alpha \sin\alpha}{-\cancel{2}\sin 4\alpha \sin(-\alpha)} = \frac{\cos 4\alpha\,\cancel{\sin\alpha}}{+\sin 4\alpha\,\cancel{\sin\alpha}} = \cot g\, 4\alpha$$

$-\sin(\alpha)$