

FORMULE DI ADDIZIONE E SOTTRAZIONE



M4010

DOMANDA : $\sin(\alpha + \beta) = \sin \alpha + \sin \beta$ NO

$$\sin\left(\frac{\pi}{2} + \frac{\pi}{2}\right) = \sin\frac{\pi}{2} + \sin\frac{\pi}{2} = 1 + 1 = 2$$

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DIMOSTRIAMO LA FORMULA $\cos(\alpha - \beta)$

$$\alpha > \beta$$

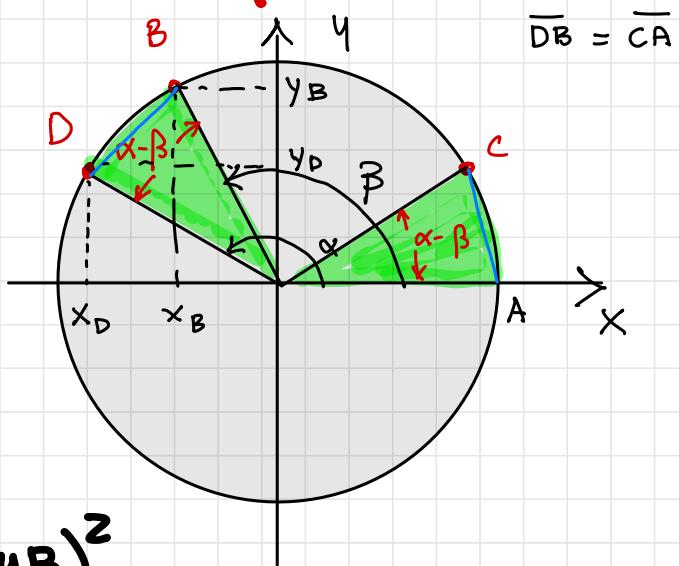
DISTANZA FRA DUE PUNTI

$$\overline{AB} = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2}$$

$$\overline{DB} = \sqrt{(x_D - x_B)^2 + (y_D - y_B)^2}$$

$$\overline{DB}^2 = (x_D - x_B)^2 + (y_D - y_B)^2$$

$$\overline{DB}^2 = (\cos \alpha - \cos \beta)^2 + (\sin \alpha - \sin \beta)^2$$



$$\overline{CA} = \sqrt{(x_c - x_A)^2 + (y_c - y_A)^2}$$

$$\overline{CA}^2 = (x_c - x_A)^2 + (y_c - y_A)^2 = [\cos(\alpha - \beta) - 1]^2 + [\sin(\alpha - \beta) + 0]^2$$

SE $\overline{DB} = \overline{CA}$ conde gli archi uguali

$$\overline{DB}^2 = \overline{CA}^2$$

$$(\cos \alpha - \cos \beta)^2 + (\sin \alpha - \sin \beta)^2 = [\cos(\alpha - \beta) - 1]^2 + \sin^2(\alpha - \beta)$$

$$\omega^2 \alpha + \omega^2 \beta - 2 \cos \alpha \cos \beta + \sin^2 \alpha + \sin^2 \beta - 2 \sin \alpha \sin \beta =$$

$$= \omega^2 (\alpha - \beta) + 1 - 2 \cos(\alpha - \beta) + \sin^2(\alpha - \beta)$$

$$1 + 1 - 2 \cos \alpha \cos \beta - 2 \sin \alpha \sin \beta = 1 + 1 - 2 \cos(\alpha - \beta)$$

$$-2 \cos \alpha \cos \beta - 2 \sin \alpha \sin \beta = -2 \cos(\alpha - \beta)$$

~~$$-2 (\cos \alpha \cos \beta + \sin \alpha \sin \beta) = -2 \cos(\alpha - \beta)$$~~

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

Funzione	Formula di addizione	Formula di sottrazione
seno	$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$	$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$
coseno	$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$	$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$
tangente	$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \cdot \tan \beta},$ $\text{con } \alpha + \beta \neq \frac{\pi}{2} + k\pi, \alpha \neq \frac{\pi}{2} + k_1\pi, \beta \neq \frac{\pi}{2} + k_2\pi$	$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \cdot \tan \beta},$ $\text{con } \alpha - \beta \neq \frac{\pi}{2} + k\pi, \alpha \neq \frac{\pi}{2} + k_1\pi, \beta \neq \frac{\pi}{2} + k_2\pi$

$$\text{ESEMPI: } \cos 15^\circ = \cos(45^\circ - 30^\circ)$$

$$\cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$$

$$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

$$\sin 165^\circ = \sin(135^\circ + 30^\circ) = \sin 135^\circ \cos 30^\circ + \cos 135^\circ \sin 30^\circ$$

$$= \frac{\sqrt{2}}{2} \frac{\sqrt{3}}{2} + \left(-\frac{\sqrt{2}}{2}\right) \cdot \frac{1}{2} = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$\sin\left(\alpha + \frac{2}{3}\pi\right) - \cos\left(\frac{\pi}{6} + \alpha\right) =$$

$$= \sin\alpha \cos\frac{2}{3}\pi + \cos\alpha \sin\frac{2}{3}\pi - \left(\cos\frac{\pi}{6} \cdot \cos\alpha - \sin\frac{\pi}{6} \sin\alpha \right)$$

$$= \sin\alpha \left(-\frac{1}{2}\right) + \cos\alpha \cdot \frac{\sqrt{3}}{2} - \left(\frac{\sqrt{3}}{2} \cos\alpha - \frac{1}{2} \sin\alpha\right)$$

$$= -\frac{1}{2} \sin\alpha + \frac{\sqrt{3}}{2} \cos\alpha - \frac{\sqrt{3}}{2} \cos\alpha + \frac{1}{2} \sin\alpha = 0$$